essence of the story I'm not going to talk, as a physicist typically would, in terms of a genuine atomic phenomena such as 'electron spin'. Instead, I'll keep things deliberately unspecific and talk simply of stuff (quantum stuff!) which can have just two properties, colour (blue or red) or shape (circle or square)[3]. Just because the presentation is cast in these less technical terms, rest assured that in no sense will I be dumbing down the treatment. I'm going to give it to you straight in a series of 'lessons'. Here comes the first!

2. Quantum mechanics Lesson 1 - Measurement and Preparation

Corresponding to the two properties of colour and shape, we have available to us two kinds of measurement, two experimental tests, which we can use to determine 'values' of the respective properties, blue or red in the case of colour, circle or square in the case of shape. If we test any piece of quantum stuff for colour then it will always produce the result red or blue. Likewise, conducting a shape measurement on a piece of quantum stuff we'll find either circle or square. Given a piece of quantum stuff of unknown origin we cannot say which of the two possible outcomes we'll obtain for a given measurement. However, if we perform two identical tests one after the other then we will always obtain identical results. For example, if we test a piece of quantum stuff for colour and find red and then immediately test for colour again, we are certain to obtain the result red. Similarly if we tested a piece of quantum stuff for shape and found square and then repeated the shape test on the same quantum stuff, then, with 100% certainty, the result would again be square. It is therefore said that by testing a piece of quantum stuff of unknown origin for colour or shape, by performing on it the respective measurement, we have **prepared** it in a known state of colour or shape respectively.

So far so sensible... Now let's do something a bit different. Suppose we prepare, as described, a large number of samples in states of known colour. That is, given a large collection of pieces of quantum stuff, all of unknown origin, we test each for colour and collect the red and blue samples separately into two subsets of known colour. Then the result of immediately repeating a colour test on a representative of either subset is known with certainty. But let's instead test, say, each of the blue subset for *shape*. We find that for about half of the samples the shape test gives the result circle with the other half of course giving the result square. Likewise for the red subset, there's roughly a 50:50 split between circle and square when a shape test is applied to each sample. In other words, given a piece of stuff prepared in a state of known colour a shape measurement is as likely to find that its circle as square. This is interesting. With the quantum stuff in a *known* state of colour I know the *probability* of finding it to be circle or square.

Let's now take a particular piece of quantum stuff which has been subjected first to a colour and then a shape test. Suppose the colour test revealed it to be red and the subsequent shape test result was square. Does this particular piece of quantum stuff then have both these attributes? Is it both square and red — a red square? Quite reasonably, you would think this must obviously be true and that if we now retest for colour we must surely confirm the earlier colour result, red. In fact this is *not* the case. There is now a 50% chance of it testing red and a 50% chance of it testing blue! The fact that it originally tested red for colour is *irrelevant*! The intervening shape measurement has caused us to lose all information about its colour. Likewise, if some quantum stuff was in a known state of shape and then subjected to a colour measurement. The colour measurement

[3]The idea of using colour and shape comes from a nice article by Frank Wilsek, "Entanglement Made Simple" available at `https://www.quantamagazine.org/20160428-entanglement-made-simple/`

would prepare it in a known state of colour, but the outcome of a subsequent shape measurement would be perfectly random. Shape and colour are an example of a pair of **complementary** properties. It is a fundamental fact of quantum mechanics that complementary properties cannot *simultaneously* be known. As a matter of principle, there is no test available to us that can simultaneously reveal both these properties. This is of course very different to what we're used to. What we could call 'classical' stuff, the objects of our direct physical experience, have properties which exist simultaneously and independently of whether we test for them. At those scales, red squares exist.

What it means for quantum stuff to have 'properties' is profoundly different from what it means for classical stuff to have properties. Physicists talk of the **state** of a system by which they mean all the information that's needed to completely describe the system. Since classical stuff can have both colour and shape its state could simply be described by a list of the values of its possible properties. For example,

$$(\bullet, \square), \tag{1}$$

could describe some classical stuff having colour red and shape circle. The properties of quantum stuff may, as is the case for our colour and shape, be complimentary and so cannot be simultaneously known. The symbolic representation of quantum stuff is necessarily different. The sort of symbol used to represent a piece of quantum stuff might look something like,

$$|\Psi\rangle\,.$$

It's called a **state vector** and is very different from the simple list of properties we used to describe a piece of classical stuff. As we've seen, known shape implies no knowledge, no information, about colour and vice versa. In the quantum world we have seen that one and the same state may correspond both to the information that "the probability of the result of a colour test being red is 1" and to the information that "the probability of the result of a shape test being square is 1/2 and being circle is 1/2". So quantum stuff requires a very different kind of representation and then very specific rules to extract from that representation meaningful information. Consequently there is a deep and apparently necessary indirectness in the quantum description which one way or another accounts for the conceptual issues we face when trying to interpret what quantum theory is telling us about the world. Whilst we have a wonderfully successful mathematical formalism for extracting information from $|\Psi\rangle$ the status of exactly what $|\Psi\rangle$ *is*, is still being debated 100 years after the the theory's original formulation.

In our case, having tested some quantum stuff for colour and found red we have prepared the stuff in a state we could denote,

$$|\bullet\rangle$$

or, if we found blue, the quantum stuff would be prepared in the state,

$$|\bullet\rangle\,.$$

Similarly if we performed a shape test on some quantum stuff we would prepare it in either the circle state,

$$|\square\rangle\,,$$

or the square state,

$$|\square\rangle\,.$$

The remarkable fact that, when we prepare some quantum stuff in a state of definite colour and then test for shape, there's a 50% chance of finding circle and 50% chance

of finding square is encoded in the following symbolics.

$$|\bullet\rangle = \frac{1}{\sqrt{2}}\left(|\square\rangle + i\,|\square\rangle\right)$$
$$|\bullet\rangle = \frac{1}{\sqrt{2}}\left(i\,|\square\rangle + |\square\rangle\right)$$
$$|\square\rangle = \frac{1}{\sqrt{2}}\left(|\bullet\rangle + |\bullet\rangle\right)$$
$$|\square\rangle = \frac{1}{\sqrt{2}}\left(|\bullet\rangle + i\,|\bullet\rangle\right)$$

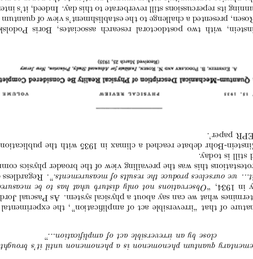
What these equations are 'saying' is that a state of known colour can be viewed as a **superposition** of shape states (the first two equations) and that likewise a state of known shape can be viewed as a superposition of known colour states (the third and fourth).[4] The important take away here is this notion of superposition. It means that two potential types of shape are simultaneously present 'in' the colour state and therefore sometimes when we test a state of known colour for shape we find circle and sometimes square. The numbers here ensure that, using the aforementioned rules for extracting information, the respective probabilities are exactly 1/2. It is very important to appreciate that if some quantum stuff is in such a superposition, *say* of circle and square states, it is *not* the same as it having *either* the shape property circle *or* square. To understand this, suppose we have a large number of pieces of quantum stuff in the known colour state red and another large number of pieces of quantum stuff which consist of a 50:50 mixture of circle and square pieces. It is certainly true that testing for shape we could not distinguish between these two collections — in both cases we'd estimate that half were square and half were circle. However, testing for colour does distinguish between them since the collection of pieces of quantum stuff which are superpositions are all in the known colour state red and so a colour test will have certainly have outcome red for each of those pieces whilst testing the 50:50 mixture of circle and square pieces will result in roughly a 50:50 split of red and blue outcomes. The reason being that each state of known shape is itself a superposition of red and blue states!

So, to reiterate, given some arbitrary piece of quantum stuff in some unknown state $|\Psi\rangle$, if we carry out a colour measurement and find the outcome red then we have prepared the piece of stuff in the state $|\bullet\rangle$. The outcome of another colour measurement on this piece is now clear — the mathematical representation of the state of the stuff is telling us it's red and testing for colour can only give this result! But if we test for shape then what's relevant is that this state has the quantum mechanical description as a superposition,

$$|\bullet\rangle = \frac{1}{\sqrt{2}}\left(i\,|\square\rangle + |\square\rangle\right).$$

The outcome of a shape measurement is therefore random — both possibilities exist in the red state with equal probability. Perhaps we test for shape and find circle, then we have simply prepared the piece of quantum stuff in the state $|\square\rangle$. The information that it once had known colour is lost forever. It's worth pausing a moment here to reflect on just how different this behaviour is to that of our common experience. You might ask, but how can that be? I'm afraid I have no answer. This is how nature responds when we probe its microstructure in our macroscopic terms, that is, using our macroscopic laboratory equipment.

[4]Don't worry about the square root of 2 and the i's appearing here — they're just there because as promised I'm giving it to you straight, exactly as a physicist would write it.



> *"Einstein darmed Bohr mit dolor dammed Bonkers ... Einstein fist mating thee, bot mating is ong a word mat ne'ther to morn mot ne'ther ther right way is to see that word"*

Alistair Zellicoff^a describes Einstein and Bohr as holding quite distinct views of the nature of physics. 'To Einstein, physics is about "what is", while for Bohr, physics is about "what we can say". Bohr's position is limited to the macroscopic world of our experience. When we want to learn and share information about quantum phenomena we must do so by supposing on the quantum world some macroscopic measurement apparatus which then defines what it is we can say about the quantum world. According to Wheeler,

> *"No elementary quantum phenomenon is a phenomenon until it's brought to a close by an irreversible act of amplification..."*

It's the nature of that "irreversible act of amplification", the experimental setup, which Bohr sees the emphasis in the Copenhagen interpretation of Bohr's physics is attempting to do. Einstein's attempts to undermine this emphasis culminated in the famous 1935 paper by Einstein, Podolsky and Rosen. Can a quantum mechanical description of physical reality be considered complete? Let's list the assumptions that lead to the conclusion that it cannot be.

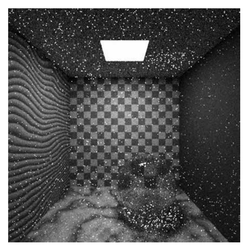
So, to the approximate question of Einstein, Podolsky and Rosen. Can a quantum description of physical reality be considered complete? The answer to this question is the crucial issue. In the paper they define a complete theory as one exactly when the conclusion means. In the paper they define a complete theory as one in which "*every element of physical reality must have a counterpart in the physical theory*". Their aim of course is to demonstrate that quantum theory is *not* complete by this criterion so we had better be able to identify both an 'element of physical reality' and whether or not it has a "counterpart in the physical theory". They provide the following test for recognising an element of physical reality.

> *"If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."*

It is a physical system in which being observed. By testing some quantum stuff for colour we may prepare it in a particular state of colour. We would then know with certainty the result of a subsequent colour measurement. Thus, in Einstein's words, there exists an element of physical reality corresponding to the colour of quantum stuff so prepared and indeed in the mathematical formalism the state is described by a state vector which we label for the known colour, $|\bullet\rangle$ or $|\bullet\rangle$. The element of physical reality then is the counterpart in the formalism. But recall that in this state we have no knowledge of the shape of our stuff and hence for quantum stuff

Notice how it was essentially ignored for the first 30 or so years while interest only picked up at the beginning of the 70s. For a paper which has cited by thing so important this is very unusual. The interest in the physics community would sometimes become assimilated into the shared body of knowledge. Shortly we'll see Einstein's most cited paper — a remarkable, perhaps even precise, key given the scale of its contribution to modern physics.

It's the nature of that "irrevocable act of amplification", the experimental setup, which Bohr sees the emphasis in the Copenhagen interpretation of Bohr's physics is attempting to do. Indeed, it's interesting to note the physics community's extraordinary subsequent reaction to the paper we mentioned by citations per year since publication.



Alice⟩(colour)	Bob⟩(Shape)	Alice⟩(Shape)	Bob⟩(colour)
●	○	○	●
●	□	□	●
●	□	□	●
●	○	○	●

Alice⟩(colour)	Bob⟩(colour)	Alice⟩(Shape)	Bob⟩(shape)
●	●	□	□

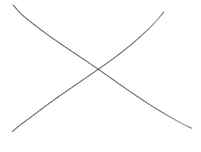
Notice that Bob has to test for colour and finds him out on piece of a pair, Bob testing the other piece of the pair for shape and finding out either circle or square etc. There is no correlation. But, when they both happen to perform the same type of measurement on each piece of the outcomes are always perfectly correlated. In particular, when Bob travels to Australia, whilst Alice travels to Alphacentauri, he can still find their ultimate explanations in quantum mechanics. Its accuracy is unprecedented. There has been no experiment to date which has found a result that is at variance with the theory over the many years it has been subjected to ever more ingenious and subtle experimental tests and its predictions have always been confirmed. Upon its validity rests the huge edifice of technology which defines modern life— it led directly to the invention of the transistor, a device which one way or another is behind all modern electronics. There would be no lasers without quantum mechanics nor would there be the nuclear magnetic resonance imaging technology so important in modern medicine. I could go on. In summary, quantum mechanics works!

So where's the weirdness? Well for one thing we're used to being able to visualise what's going on in our physical theories. But taking literally the quantum mechanical description of the atomic world, applying to it the visual grammar of our intuition, we find ourselves mired in a philosophical house of mirrors, encountering paradoxes at every turn. The problems arise from certain peculiar features of the quantum world, features which do not appear to have counterparts in the world of our direct physical experience. Perhaps the deepest of these is the notion of *superposition*, the central mindfulness is at work, or lost in the sense that the answers to questions we can ask of the quantum world are in general and in principle random. This randomness seems not to have to do with a lack of complete knowledge of some deterministic mechanism at work 'behind the scenes', but is an inherent feature of how we are able to interface with the microworld. Another curious feature of the quantum formalism is the indirect way in which the current knowledge of a system, it's 'state', is encoded mathematically. This quantum mechanical 'state vector' is the central object of study and yet its status, beyond that of a calculational device, is still hotly debated.

We'll explore these and other strange aspects of the quantum world and its formalism in what follows. I want to give as honest an account as possible of what makes quantum mechanics weird using essentially no mathematics but I also want to show you how physics is done. The physicist's job is not to worry about how to give you a genuine look at their symbolic language. So as not to obscure the

[1]I'll interchangeably use the terms quantum mechanics, quantum theory and quantum physics.

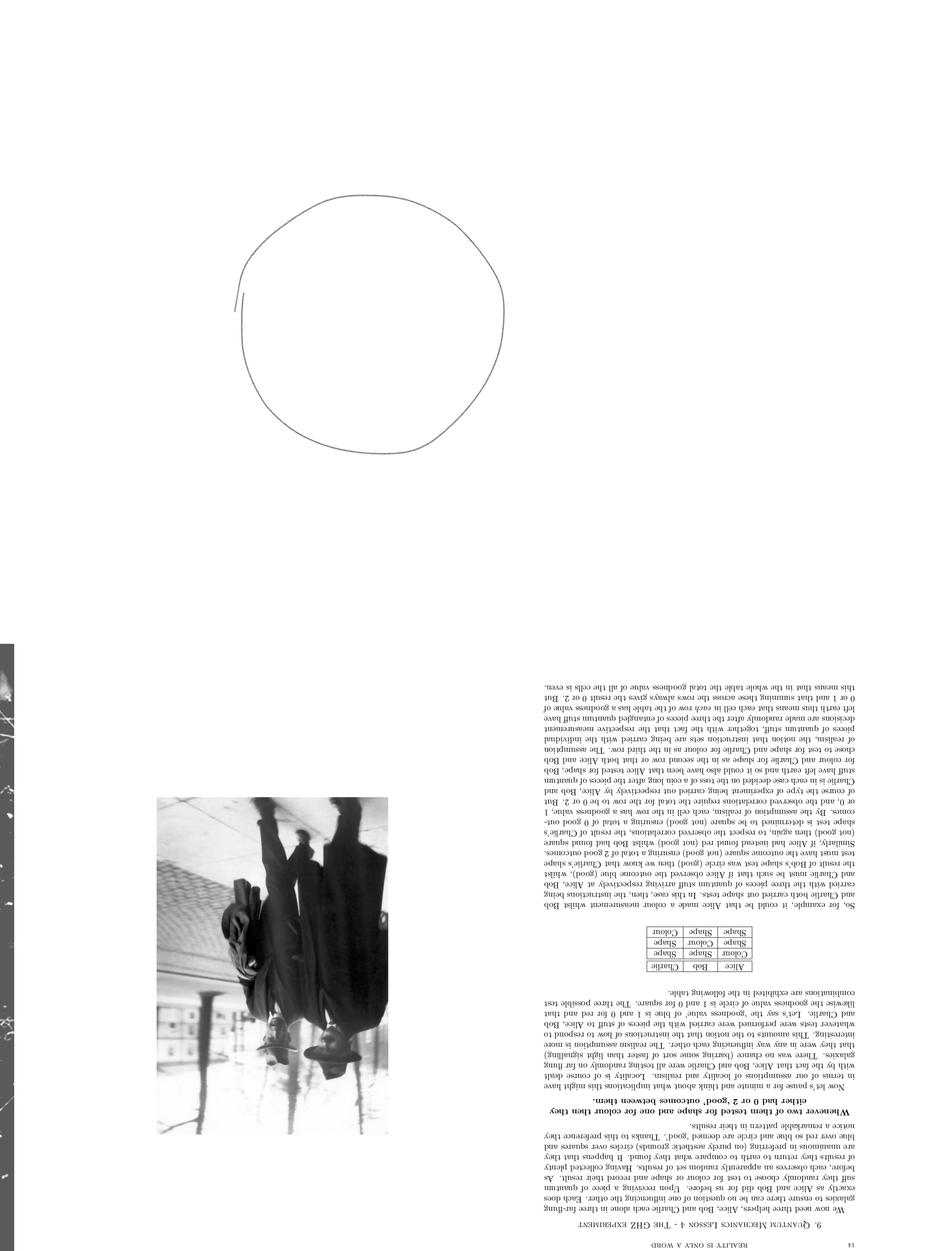
[2]In truth, quantum mechanics is regarded as nothing less that the correct framework for *all of* physics.



Pre-quantum physics had been about establishing a supplied between the real world of our experience and a mathematical infrastructure. The physics unfolding revealed all aspects of the reality. Things in this scale function are describe properties, formalism and we ascribe things directly as a list of the values of those properties, (● or □). The quantum world is very different. We cannot separate properties. The world and there, from the lens we construct. A test can take us from a minimal state of knowledge, $|\Psi\rangle$, to something like (● or □) but *not* both.

3. The Einstein-Bohr debate and the EPR Paper

Niels Bohr and Albert Einstein, giants of physics in early 20th century, and created intellectual history. They were friends but profound disagreement over the nature of reality we have to know. Einstein was deeply committed to the view that the universe is real in the sense that the properties of the things we observe exist independently of whether we look at them or not. Bohr



9. Quantum Mechanics Lesson 4 - The GHZ experiment

We now need three players, Alice, Bob and Charlie and adore to three far-flung galaxies to ensure there can be no question of one thinferencing the other. Each does exactly as Alice and Bob did in the before. Upon receiving a piece of quantum stuff each observes an apparently random set of results. Having collected plenty of results they return to earth to compare what they found. It happens that they have ventured to three different (as purely aesthetic grounds) cities over squares and observations in preferred (as purely aesthetic grounds) cities over squares and notice a remarkable pattern in their results.

Whenever two of them tested for shape and one for colour then they either had 0 or 2 'good' outcomes between them.

Now let's pause for a minute and think about what implications this might have with the fact that Alice, Bob and Charlie were all using perfectly normal stuff, that is, they tested the exact same property on a single pair of identical quantum stuff. Further to the fact that always agreed and that 25% of the time they'd both observe red, 25% of the time they'd both observe green, 25% of the time they'd both observe blue and perfect agreement with the experiment results in the table above.

So perhaps Einstein was right. Perhaps there's a bow in socks and the quantum mechanics is somehow missing these hidden variables working in the background. But before we get to that let's round out our discussion of entanglement by quickly unanswered for about 30 years, until in 1964 Bell made a startling discovery. But before we get to that let's round out our discussion of entanglement by quantum mechanics are underpinned by the assumption of **realism** - Einstein's insistence that physics be about 'what is'. The idea that things have properties whether or not we test for

In the discussion of the EPR argument I focused on Einstein's belief that there must exist some hidden variables accounting for the perfect correlations. This is often described as the assumption of **realism**. Einstein's insistence that physics be about 'what is'. The idea that things have properties whether or not we test for them. Now, as we'll see, this assumption is incorrect.

8. John Bell's contribution

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\bigcirc\rangle_A|\square\rangle_B - |\square\rangle_A|\bigcirc\rangle_B\right)$$

In this form we see, for example, that if Bob tests to shape and finds square then the updated description of the entangled pair is described by the state $|\square\rangle_A|\square\rangle_B$ and so if Alice measures shape she too will find square.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\bigcirc\rangle_A|\square\rangle_B - |\square\rangle_A|\bigcirc\rangle_B\right)$$

which shows us that if Alice instead tests for shape then the result is entirely random. There's a 50% chance she'll find circle and 50% chance she'll find square. We can also rewrite the original state in terms of shape states as follows (this is simply a matter of plugging in the expressions we saw earlier relating known colour states to superpositions of shape states).

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\bigcirc\rangle_A|\square\rangle_B - |\square\rangle_A|\bigcirc\rangle_B\right)$$

When they don't test for the same property there is no correlation.

Now suppose Alice tests colour and finds blue. This means that the pair of quantum stuff are described by the state $|\blacklozenge\rangle_A|\blacklozenge\rangle_B$ and we see that Alice's piece of stuff must also have colour blue. If she tests for colour she is sure to find blue. Or the other hand, this post Bob measurement state can also be written as

$$|\blacklozenge\rangle_A|\blacklozenge\rangle_B = \frac{1}{\sqrt{2}}\left(|\bigcirc\rangle_A + i|\square\rangle_A\right)|\blacklozenge\rangle_B$$

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It's like what we saw before with a state of definite colour being described as a superposition of the two possible shape states.

Now suppose Bob tests his piece of quantum stuff for colour and finds blue. This new information eliminates the bed possibility from the description of the state Alice's piece of stuff must also have blue colour. If she tests for colour she is sure to find blue. Or the other hand, this post Bob measurement state can also be written as

$$|\blacklozenge\rangle_A|\blacklozenge\rangle_B = \frac{1}{\sqrt{2}}\left(|\bigcirc\rangle_A + |\square\rangle_A\right)|\blacklozenge\rangle_B$$

which shows us that if Alice instead tests for shape then the result is entirely random. There's a 50% chance she'll find circle and 50% chance she'll find square. Alice's stuff is entirely uncorrelated with Bob's. Bob's test has no effect on Alice's piece of stuff. The overall description is a sum of two possibilities. It's a superposition. It's like what we saw before with a state of definite colour being described as a superposition of the two possible shape states.

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